

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-1/2 EXAMINATION – WINTER 2021****Subject Code:3110015****Date:21/03/2022****Subject Name:Mathematics - 2****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	Marks
Q.1 (a) Find $L\{t^3 e^{-4t}\}$.	03
(b) Find $L^{-1}\left\{\frac{6e^{-2s}}{s^2 + 4}\right\}$.	04
(c) Verify Green's theorem for the function $\vec{F} = (x + y)\mathbf{i} + 2xy\mathbf{j}$ and C is the rectangle in the xy-plane bounded by $x = 0, y = 0, x = a, y = b$.	07
Q.2 (a) Find $L\{te^{4t} \cos 2t\}$.	03
(b) Find the Fourier cosine integral of $f(x) = \frac{\pi}{2}e^{-x}, x \geq 0$.	04
(c) (i) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point (2,1,3) in the direction of $\vec{a} = (1,0,-2)$.	03
(ii) If $\vec{F} = (2y + 3)\mathbf{i} + xz\mathbf{j} + (yz - x)\mathbf{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the path	04
C: $x = 2t^2, y = t, z = t^3$ from $t=0$ to $t=1$.	
OR	
(c) Solve in series $3xy'' + 2y' + y = 0$ using Frobenius method.	07
Q.3 (a) Find the arc length of the curve (semi-circular) $x(t) = \cos t, y(t) = \sin t, z(t) = 0; 0 \leq t \leq \pi$.	03
(b) A vector field is given by $\vec{F} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2y)\mathbf{j}$. Show that \vec{F} is irrotational and find its scalar potential.	04
(c) Use divergence theorem for $\vec{F} = (x^2 - yz)\mathbf{i} + (y^2 - zx)\mathbf{j} + (z^2 - xy)\mathbf{k}$ over the surface of rectangular parallelepiped, $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ to evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$.	07
OR	
Q.3 (a) Solve $\frac{dy}{dx} - y \cot x = 2x \sin x$.	03
(b) Solve $y'' + y' - 12y = e^{6x}$.	04
(c) Solve $\frac{dy}{dt} - 4y = 2e^{2t} + e^{4t}$ by Laplace transformation.	07

- Q.4** (a) Solve $\frac{dy}{dx} + \frac{y}{x} = y^3$. **03**
- (b) Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$. **04**
- (c) Solve $y'' + 9y' = 2x^2$ using the method of undetermined coefficients. **07**
- OR**
- Q.4** (a) Solve $4xp^2 = (3x - a)^2$. **03**
- (b) Solve $x^2 y'' + xy' - 4y = x^2$. **04**
- (c) (i) Express $2 - 3x + 4x^2$ in terms of Legendre's polynomial. **03**
- (ii) Find ordinary and singular points of $2x^2 y'' + 6xy' + (x + 3)y = 0$. **04**
- Q.5** (a) Solve $(y - px)(p - 1) = p$. **03**
- (b) Solve $(D^3 + D)y = \cos x$. **04**
- (c) Solve $y'' + 4y = \sec 2x$ by using the method of variation of parameters. **07**
- OR**
- Q.5** (a) Solve $(D^3 - 6D^2 + 11D - 6)y = 0$. **03**
- (b) Solve $(2x + 3)^2 y'' - 2(2x + 3)y' - 12y = 6x$. **04**
- (c) Find the series solution of $(1 + x^2)y'' + xy' - 9y = 0$ near the ordinary point $x=0$. **07**

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